

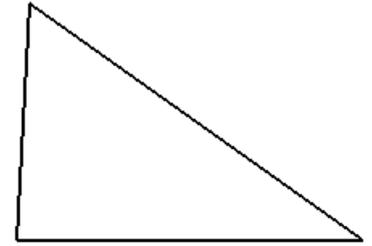
Understanding Geometric Constructions

Note: all sketches are created in the drawing environment of SolidWorks

Bisecting a line

The geometrical construction for bisecting a line is based on the geometry of an **isosceles triangle**.

In order to investigate this we will first sketch any triangle and add relations to create an isosceles triangle.



What are the characteristics of an isosceles triangle?

An isosceles triangle has two equal sides and two equal angles

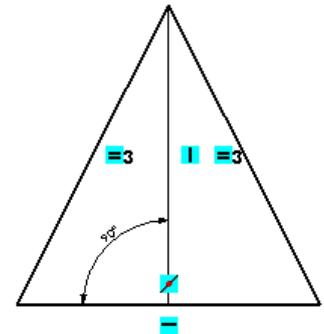
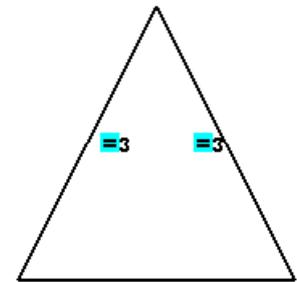
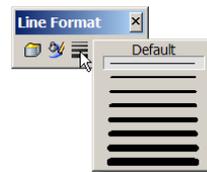
Add the relations shown opposite to create the isosceles triangle.

Sketch a line ensuring one endpoint is coincident with the apex and the other coincident with the midpoint of the base of the triangle.

Smart dimension the angle as shown. Accept “*Make the dimension driven.*”



Line weights may be edited increased or decreased by selecting the line and choosing an appropriate line weight from the line format toolbar.

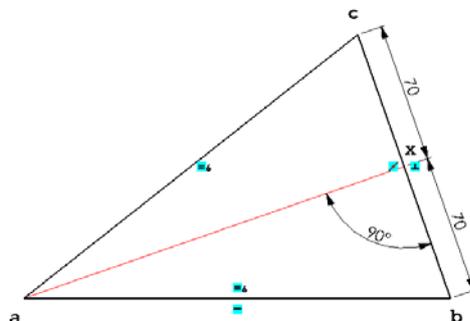


The line drawn from the apex of the triangle to the midpoint of the base makes an angle of 90° with the base.

This also works in reverse; a line drawn from the apex of the isosceles triangle perpendicular to the base bisects the base.

In general terms;

In an isosceles triangle, a line drawn from the intersection of the two equal sides perpendicular to the opposite side, will bisect that side.

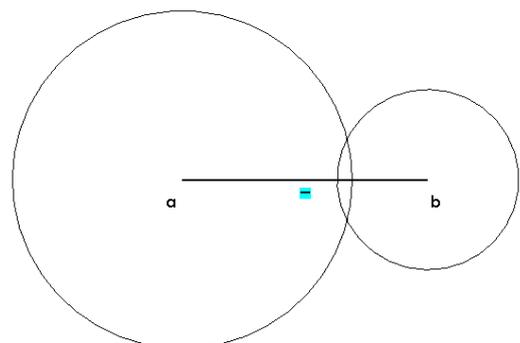


$$\angle axb = 90^\circ$$

$$|cx| = |xb|$$

We will now investigate the geometrical construction used in bisecting a line based on this principle.

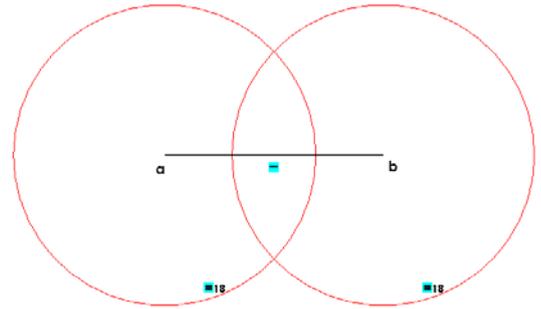
Create the sketch shown opposite. The centres of the circles are located on the endpoints of the line.



Add an equals relation between the circles.

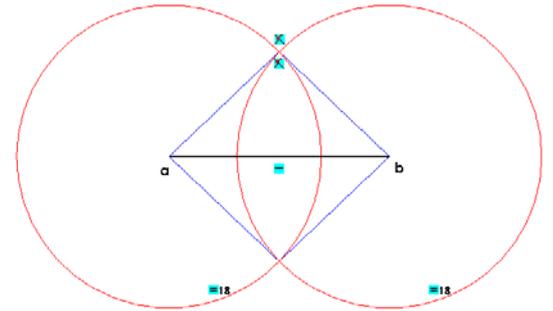
 Drag the circumference of the circle if necessary to ensure that the circle circumference intersect one another.

 Line colour may be edited on the line format toolbar.



Add the lines shown, coincident with the endpoints of ab and the intersection of the circles.

Because the blue lines are the radii of the circles and the circle radii are equal, 2 isosceles triangles are formed.



Add a line from the apex of the isosceles triangle perpendicular to ab . This line intersects ab at the midpoint. Why?

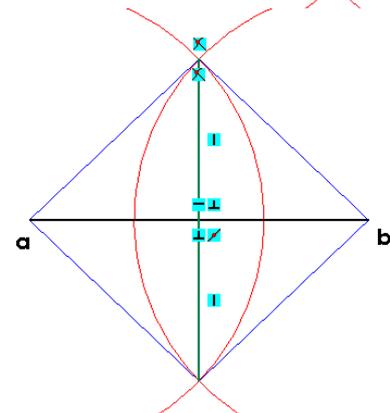
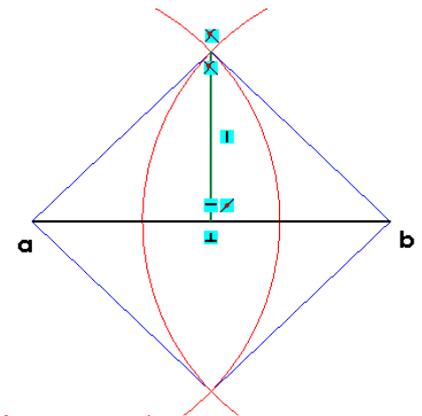
In an isosceles triangle, a line drawn from the intersection of the two equal sides perpendicular to the opposite side, will bisect that side.

Add a line from the apex of the other isosceles triangle perpendicular to ab .

Because this line is drawn from the apex of the isosceles triangle perpendicular to ab this also intersects ab at the midpoint.

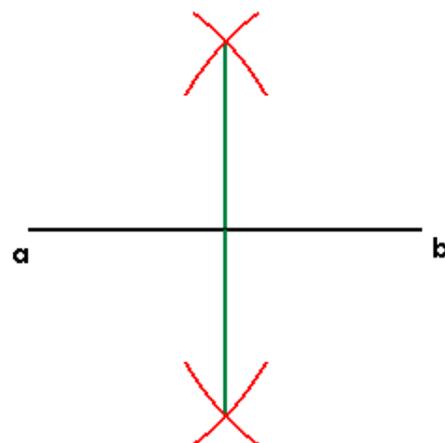
The two lines from the apex of both isosceles triangles are collinear as they are both perpendicular to ab and pass through its midpoint.

Therefore, a line from the apex of both isosceles triangles will be perpendicular to the base and will bisect it.



Geometric construction for bisecting a line.

Typically the construction appears as shown, with the arcs 'trimmed', and lines representing the isosceles triangles omitted, for simplicity of solution.



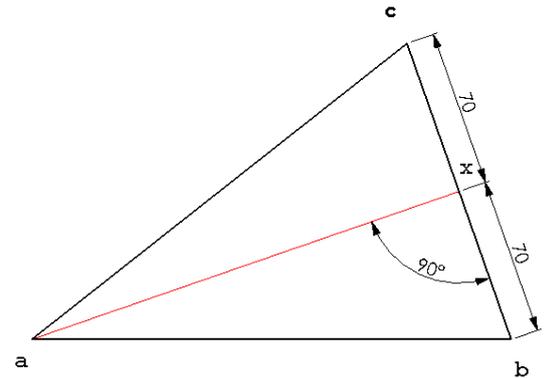
Finding the centre of a circle

The solution to determining the centre of a given circle utilises the same geometric principle as the bisecting of a line/angle.

In an isosceles triangle, a line drawn from the intersection of the two equal sides perpendicular to the opposite side, will bisect that side.

in reverse

In an isosceles triangle, a line perpendicular to the base through its midpoint, when extended, will coincide with the intersection of the two equal sides



Lets investigate!

Sketch the given circle. Add the chord as shown.



What is a chord?

A **chord** is a line segment whose endpoints lie on the circumference of a circle.

<http://mathworld.wolfram.com/Chord.html>

Add in the radii shown.

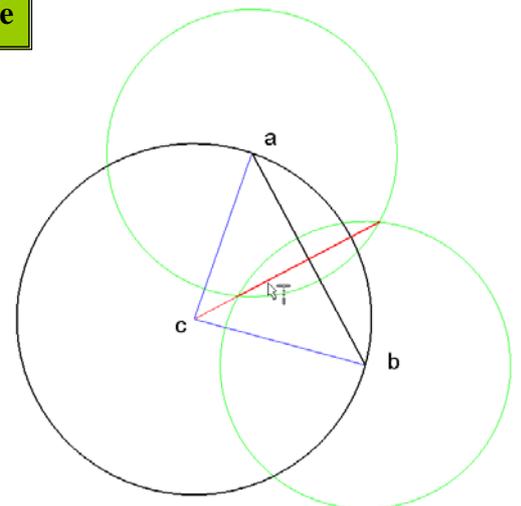
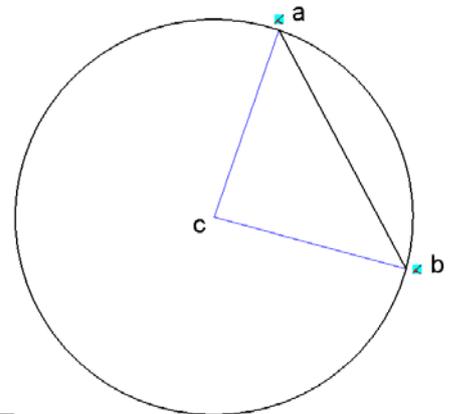
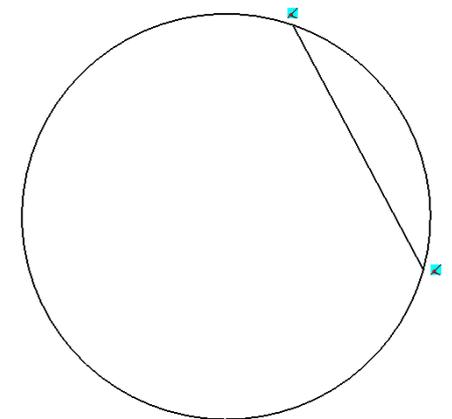
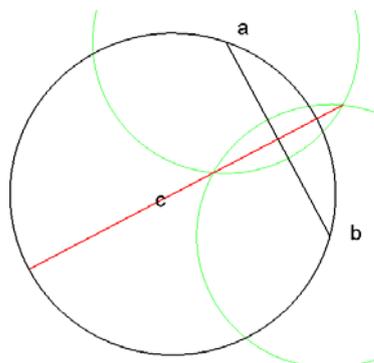
Because the two radii are equal an isosceles triangle is generated.

As we discovered in bisecting a line, a line perpendicular to ab , through its midpoint, will pass through the intersection of the two equal sides, c , the centre of the circle

Bisect ab as shown and extend the bisector to the intersection of the two radii, the centre of the circle.

The bisector of a chord of a circle will pass through the circle centre

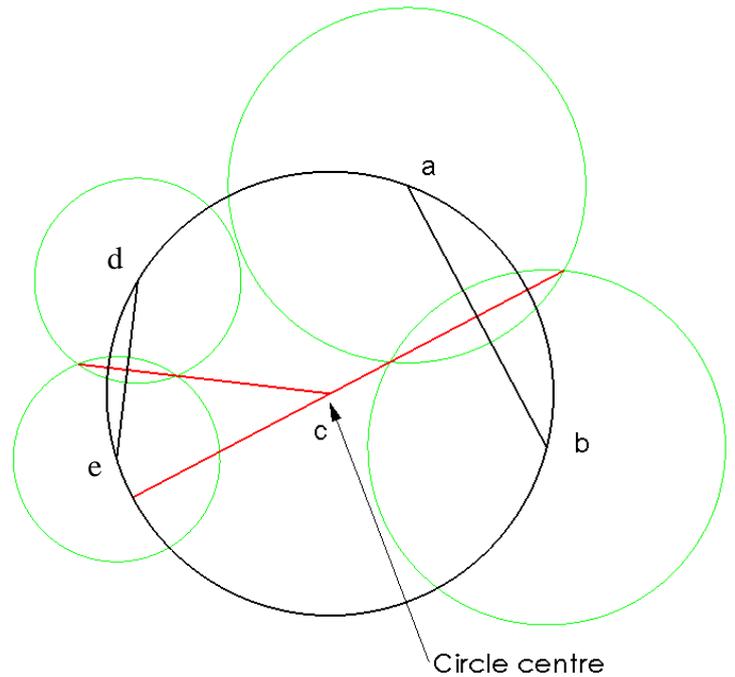
In reality the radii generating the isosceles triangle, the blue lines, will not be in place. The bisector will still pass through the centre but will not locate it, as demonstrated below.



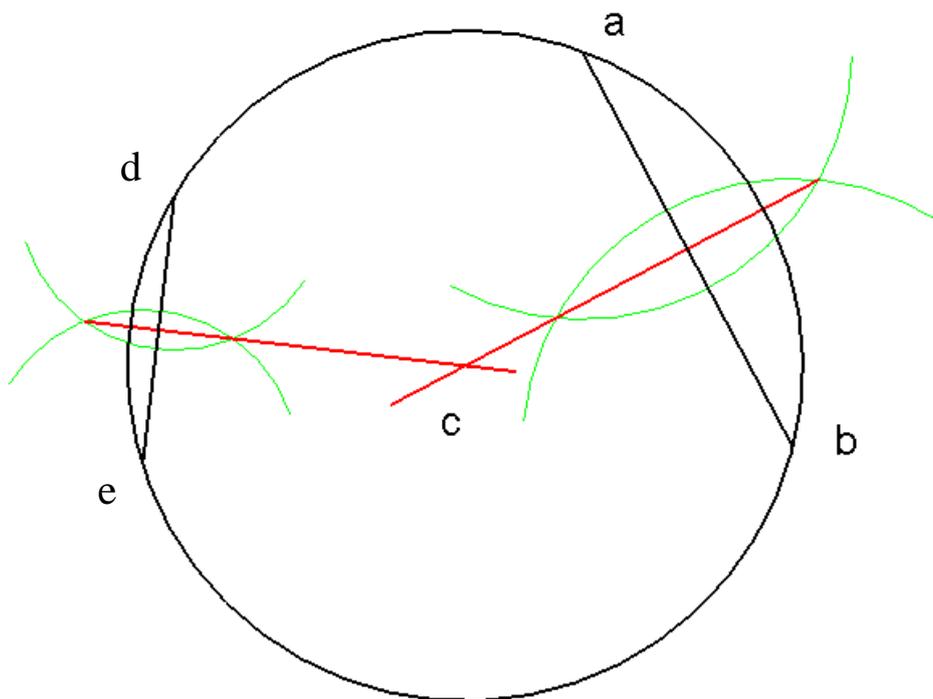
How will we find the centre?

Add a second chord, de . Bisect the chord. The bisector, when extended, will pass through the centre of the circle, as discussed previously.

The intersection of the bisectors of the two chords presents the centre of the circle, c .



In practice, the construction will appear as shown below.

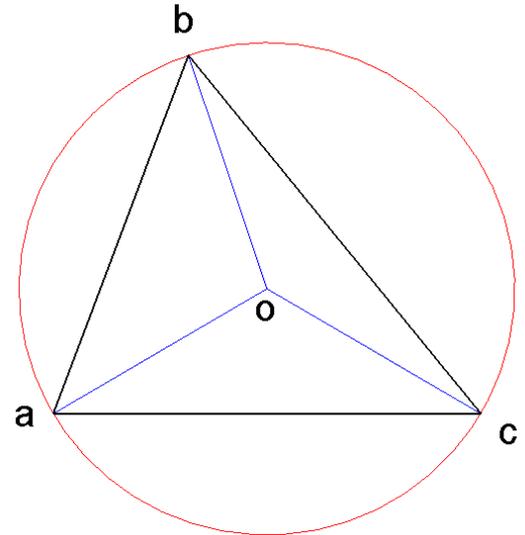


Circumscribing a circle about a triangle.

Circumscribing a circle about a triangle refers to the generation of a circle whose circumference contains all three vertices of the triangle, as shown opposite.

Given the triangle the solution requires finding the circle centre. This centre is referred to as the **Circumcentre**. Lines joining the centre of the circle to the vertices of the triangle are equal to the radius of the circle.

Three isosceles triangles are generated $\triangle aob$, $\triangle aoc$ and $\triangle cob$



All 3 points, a b and c are contained on the circumference. Therefore **ab**, **bc** and **ac** are all chords of the circle.

A chord is a line segment within a circle that touches 2 points on the circumference of the circle.

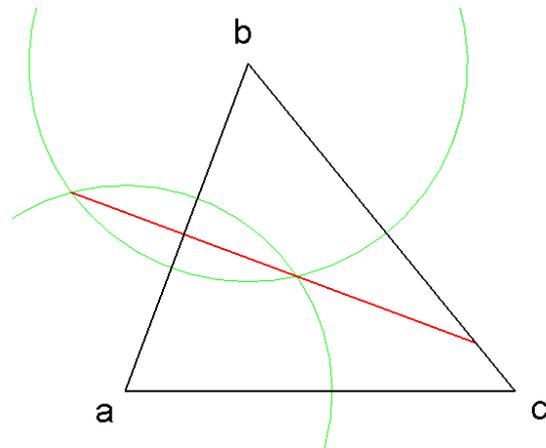
As discovered earlier in finding the centre of a circle;

The bisector of a chord of a circle will pass through the circle centre

Create a sketch similar to $\triangle abc$ shown above.

Bisect **ab**. Extend the bisector as shown.

This line contains the **circumcentre**

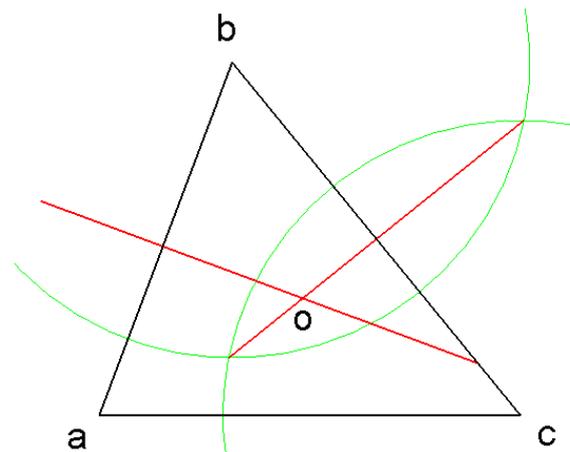


Bisect one of the remaining chords **bc** or **ac**

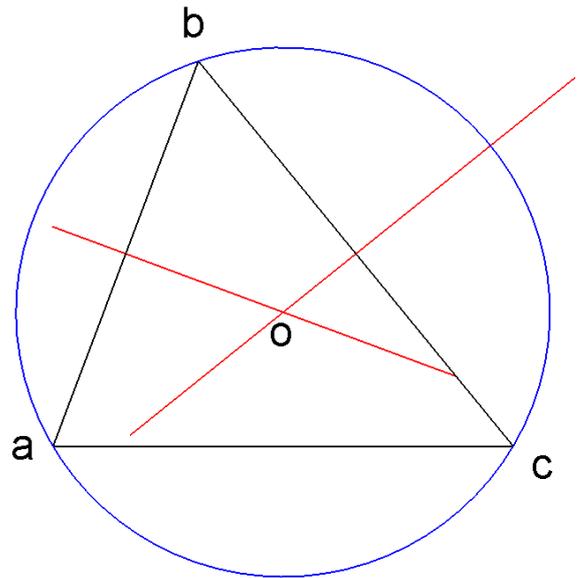
Note – The construction for bisecting **ab** has been omitted to aid clarity.

The two chords contain the **circumcentre**

They intersect at a common point **o**, the centre of the circle.

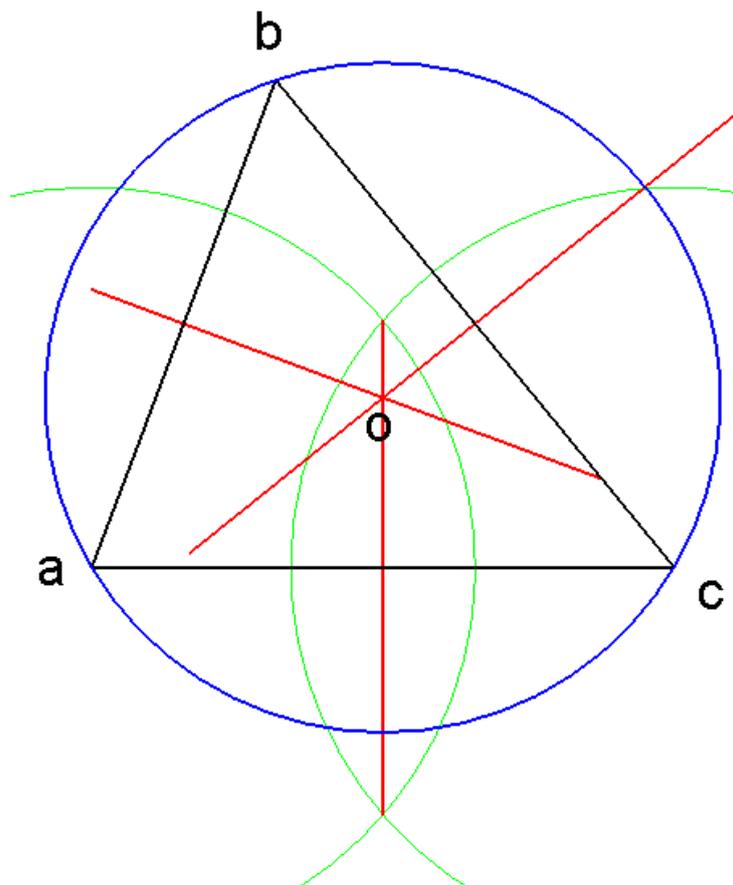


Sketch a circle with centre **o** and coincident with points **a** **b** and **c**



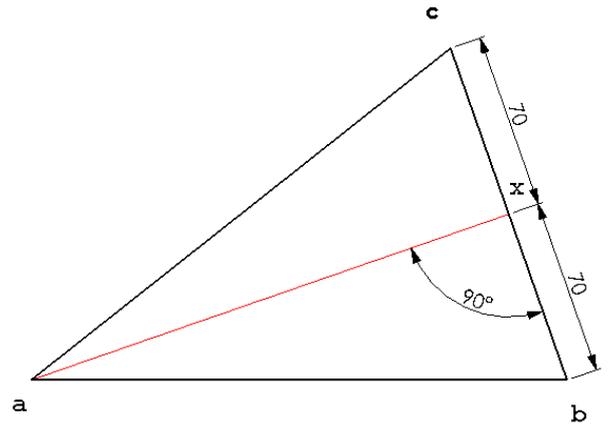
Bisect the chord **ac**.

The bisector of **ac** will also pass through point **o**.



Bisecting an Angle

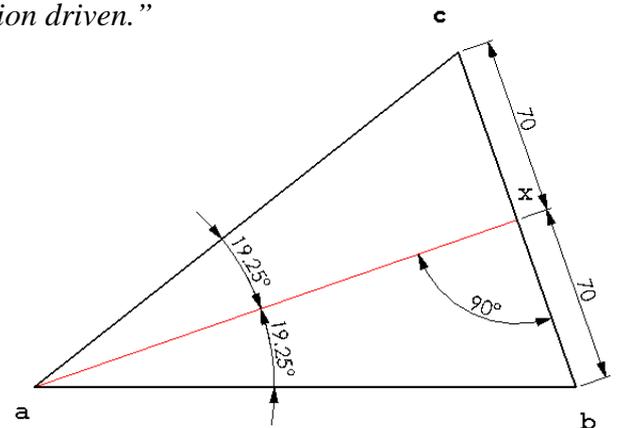
To investigate the geometrical construction for bisecting an angle we will first interrogate the model shown opposite a little further.



Smart dimension the $\angle cax$ and $\angle bax$. Accept “*Make the dimension driven.*”

The two angles are equal.

The line drawn from the intersection of the two equal sides perpendicular to the base not only bisects the opposite side but also $\angle cab$.

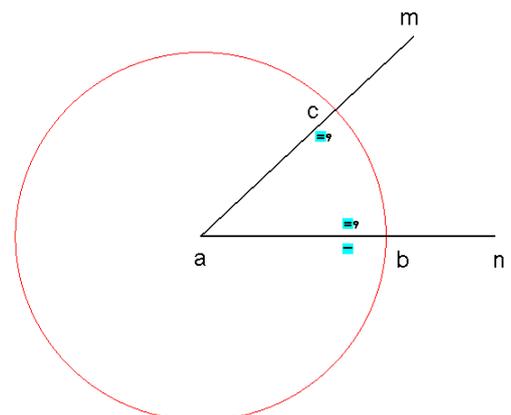


In an isosceles triangle, a line drawn from the intersection of the two equal sides perpendicular to the opposite side will bisect that angle.

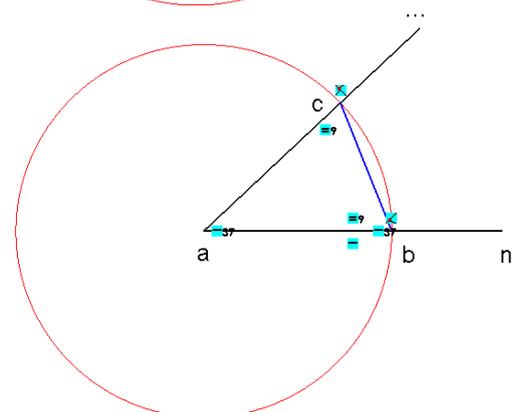
We will now examine the geometrical construction for bisecting an angle and investigate the application of this general principle.

Create the sketch shown opposite adding the necessary sketch relations.

The centre of the circle is coincident with point ‘a’.

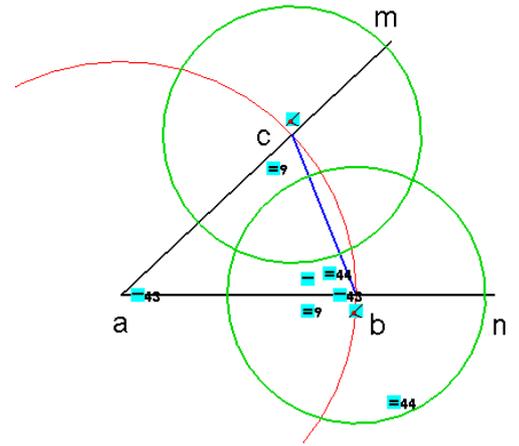


Add the blue line with endpoints coincident with the intersection of the circle and the arms of the angle.



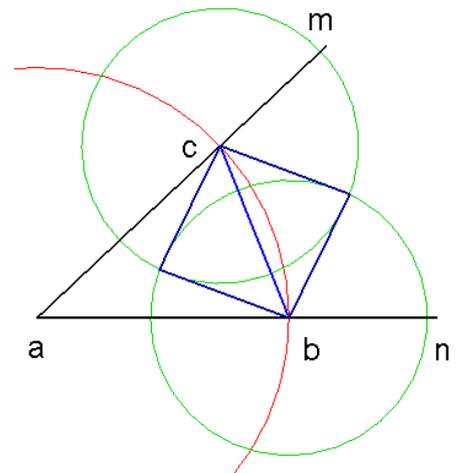
$|ab|$ and $|ac|$ are equal as they are radii of the circle, therefore $\triangle bac$ is an isosceles triangle.

Add two circles with their centre points coincident with 'a' and 'b'.
Add an equals relation between the circles.



Two further isosceles triangles are created by joining both 'b' and 'c' to the intersection of the two circles as shown.

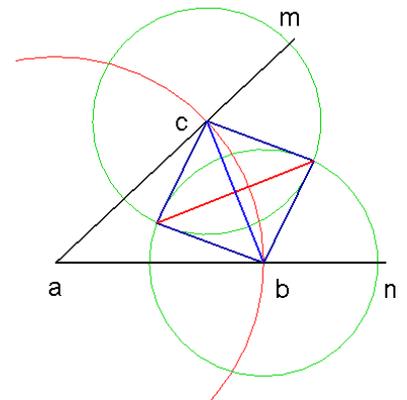
 Display of the sketch relations may be turned off by choosing **View, Sketch Relations** from the drop down menu.



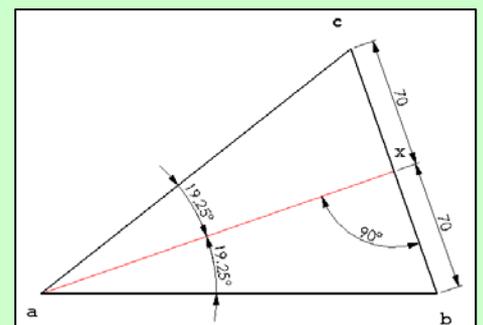
Add the red line joining the apex of both isosceles triangles.

From our investigation of the geometry associated with bisecting a line;

A line from the apex of both isosceles triangles will be perpendicular to the base and will bisect it.



In an isosceles triangle, a line drawn from the intersection of the two equal sides perpendicular to the opposite side will bisect that angle.

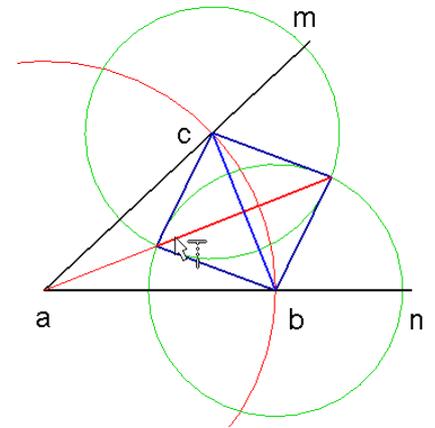
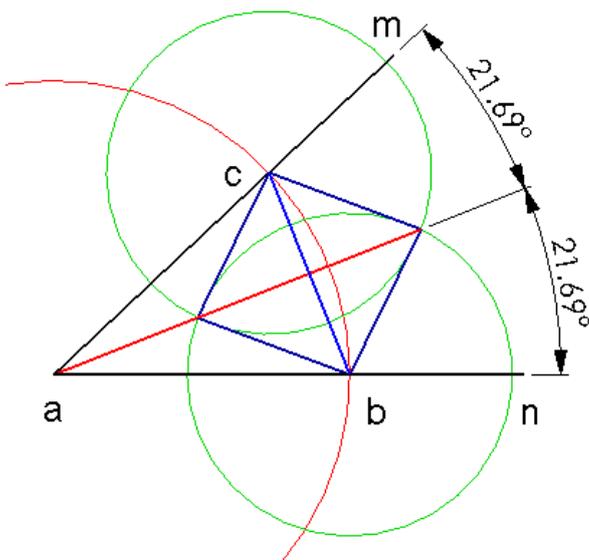


The bisector of the base passes through the midpoint and is perpendicular to, therefore, if extended, it will be coincident with point a and will bisect $\angle bac$

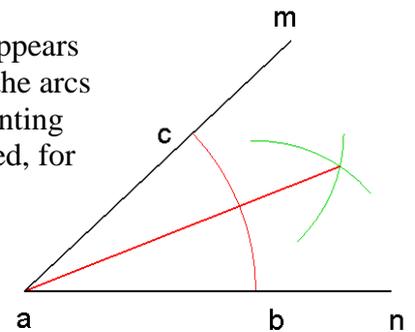
Choose **Tools, Sketch Tools, Extend...**

Move the cursor over the bisector and a preview of the extended line will be displayed. Select the line.

Smart dimension the two angles created to ensure that they are equal.



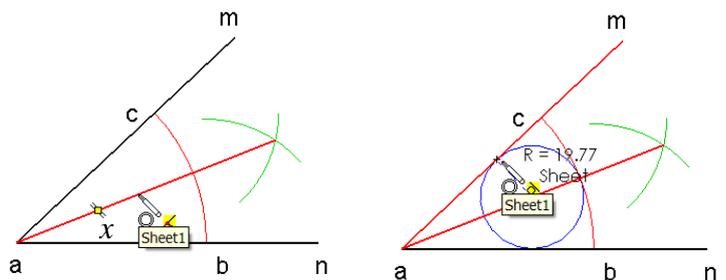
Typically the construction appears as shown on the right, with the arcs 'trimmed', and lines representing the isosceles triangles omitted, for simplicity of solution.



Further investigation;

Sketch a circle with centre x coincident with the bisector and circumference tangential to am .

The circle produced will be tangential to an also.



Draw 2 lines coincident with the circle centre and the point of contact with the arms of the angle.

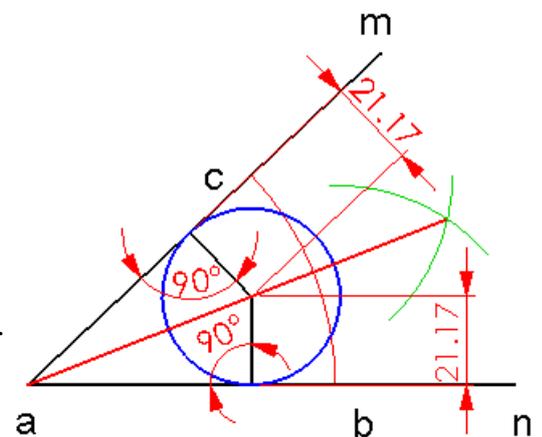
Smart dimension these two lines. - **The two lines are of equal length.**

All points on the bisector of the angle are equidistant from the arms of the angle

Smart dimension the angle between the radii and the arms of the angle.

The two angles are equal and are right angles - 90°

The arm of the angle is **tangential** to the circle. The line joining the centre to the point of contact is referred to as the **normal**. The angle between the normal and the tangent is always 90° .



Inscribing a circle in a triangle

An inscribed circle is a circle contained within a triangle which is tangential to all three sides.

The centre is referred to as the **incentre**.

Create the sketch shown opposite adding a **tangential relation** between the circle and all three sides.

Add the lines **ao**, **bo** and **co** from the vertices of the triangle to the **incentre**

Draw a line from the incentre coincident with the **point of contact** between the inscribed circle and the **tangential side**.

This line is referred to as a **normal**

Smart dimension the angle between the **tangential side** and the **normal**

The angle between a tangent and a normal is always 90°

Smart dimension the distances from the incentre to the 3 sides of the triangle.

Obviously, the 3 distances are equal, and the radii of the circle.

The incentre is equidistant from all three sides of the triangle and a distance equal to the radius away from each side.

Smart dimension $\angle cao$ and $\angle bao$

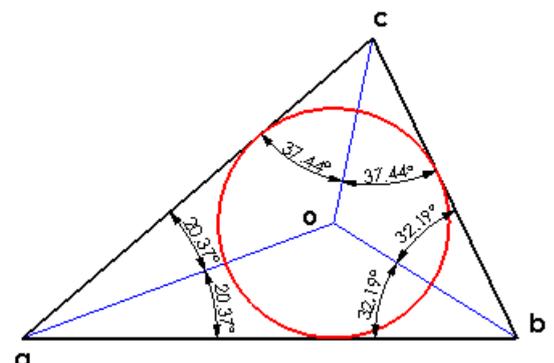
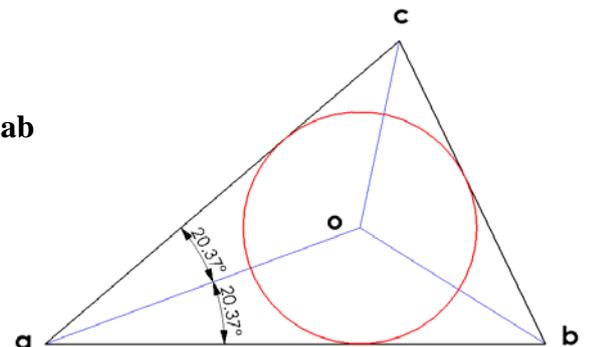
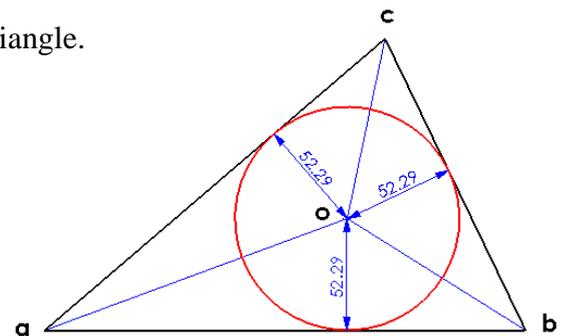
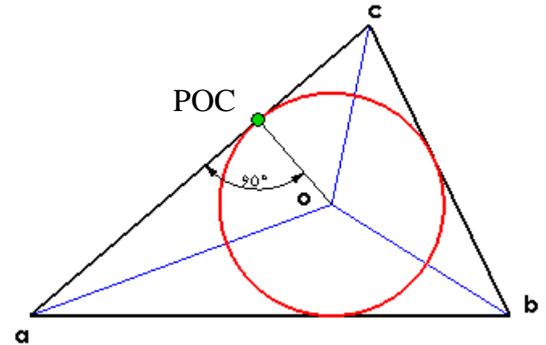
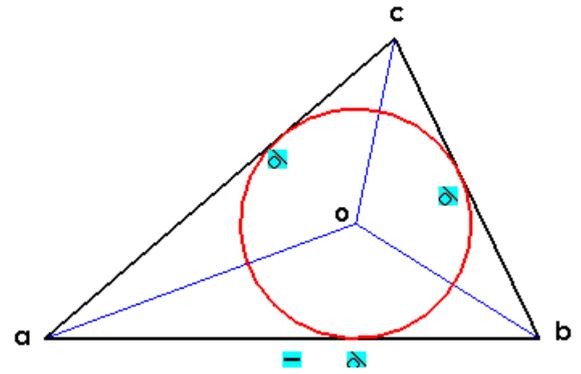
$\angle cao = \angle bao$, therefore the incentre is located on the bisector of $\angle cab$

Smart dimension the remaining angles, $\angle aco$, $\angle bco$, $\angle abo$ and $\angle cbo$

$\angle aco = \angle bco$, the incentre lies on the bisector of $\angle acb$

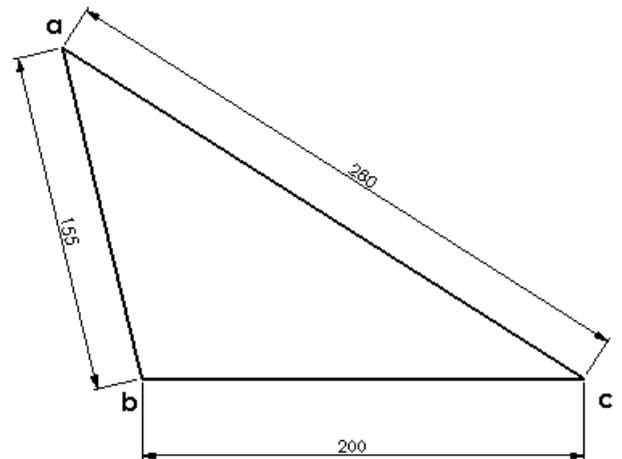
$\angle abo = \angle cbo$, the incentre lies on the bisector of $\angle abc$

The incentre is located at the intersection of the bisectors of all three angles



Given a triangle to locate the incentre, determine the radius and inscribe the circle.

Sketch the triangle shown opposite.



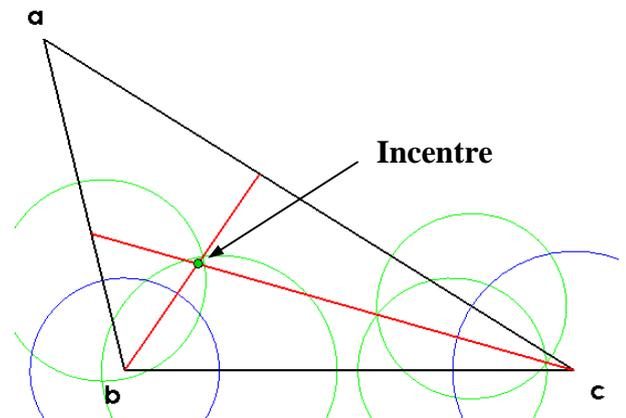
Bisect any two angles of the triangle.

The incentre is located at the intersection of the bisectors of all three angles

The incentre is located at the intersection of the two bisectors.

How do we determine the radius of the inscribed circle?

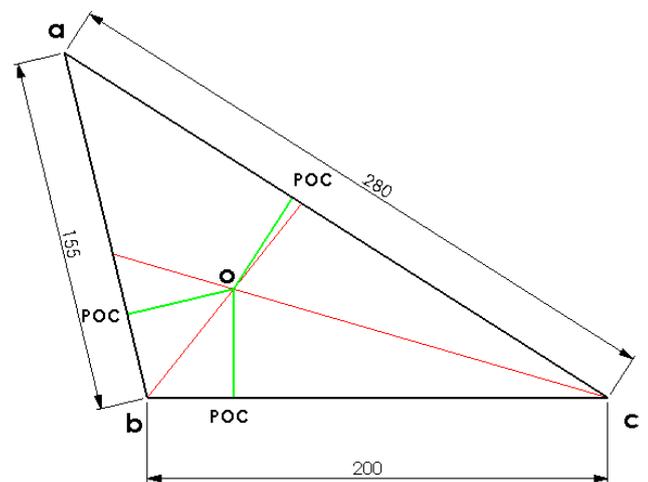
The 3 sides of the triangle are tangential to the inscribed circle.



The normal, drawn from the incentre perpendicular to the side will reveal the position of the point of contact, which is located on the circumference of the circle.

The incentre is equidistant from all three sides of the triangle and a distance equal to the radius away from each side.

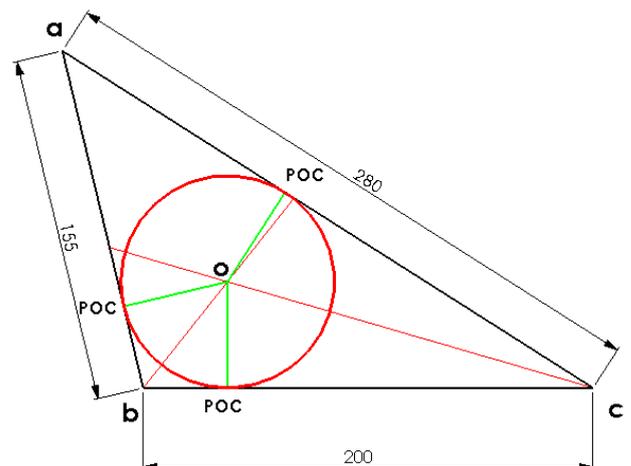
The angle between the tangent and the normal is always 90°



Add in the radii from the centre including a coincident and perpendicular relation with the sides of the triangle.

Where the normal intersects the sides yields the **POC's**

Sketch the inscribed circle with centre **o** and circumference **coincident** with the **POC's**

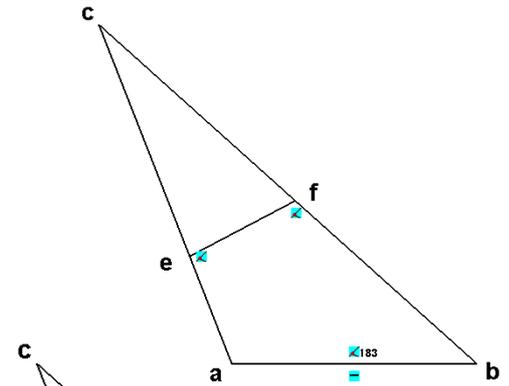


Division of Lines

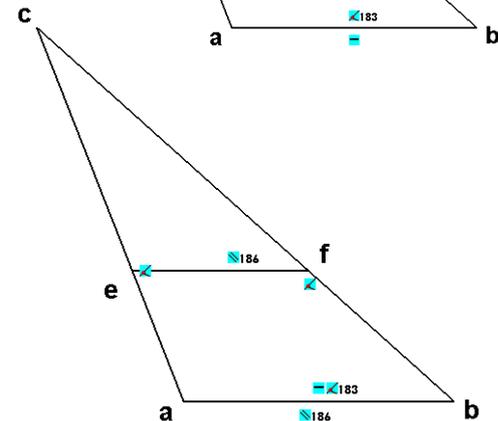
The division of lines construction bases itself on the geometrical properties of a triangle. The following text will take us through an investigation of the concepts to establish the principle involved.

Create the triangular sketch shown opposite.

Draw a line with endpoints coincident with the **ac** and **bc** to locate points **e** and **f**.

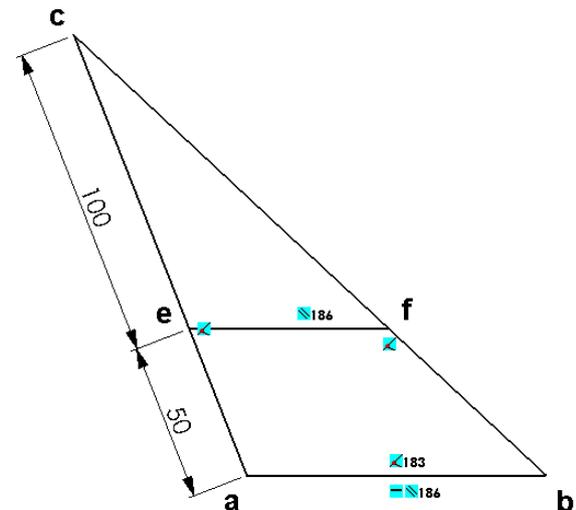


Create a parallel relation so that line **ef** is parallel with line **ab** as shown.



Smart dimension the line **ae** to **50mm** and **ec** to **100mm**.

ef remains parallel to the base **ab** due to the parallel relation constraint.



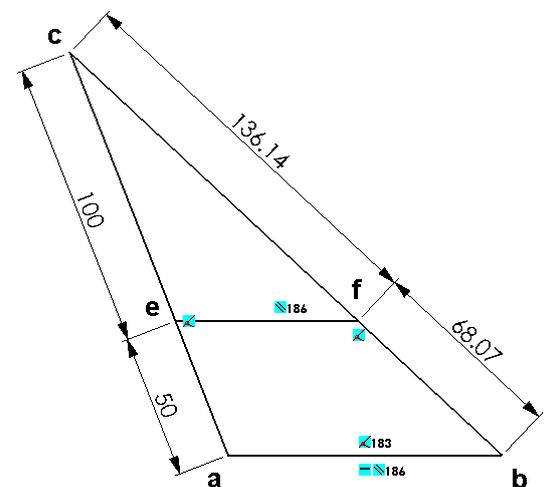
Smart dimension the remaining two sides accepting the presented dimensions. Choose *Make this dimension driven*

Taking the dimensions displayed opposite;

$ae/ec = 50/100 = 1/2$ therefore **ac** is divided in the ratio of **1:2**

$bf/fc = 68.07/136.14 = 1/2$ therefore **bc** is divided in the ratio of **1:2**

ef, which is parallel to **ab**, divides **ca** and **cb** in the same ratio



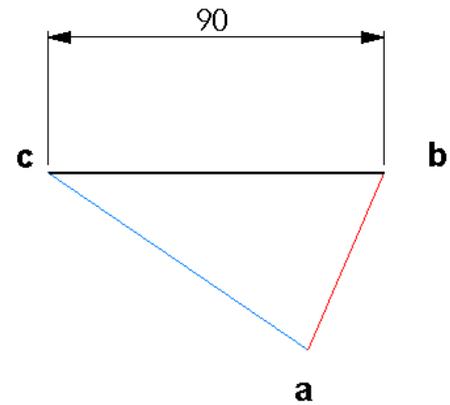
Two sides of a triangle are divided proportionally by a line drawn parallel to the third side.

We will now apply this principle to bisecting a line, dividing the line **cb** it into 2 equal parts.

Sketch the line **cb**. Smart dimension the line 90mm long.

Create the line **ca** a random length and making a random angle with **cb**.

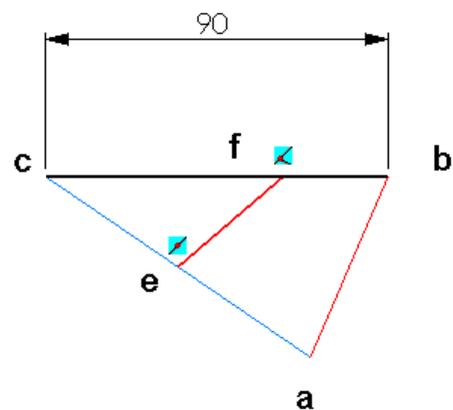
Complete the triangle by joining **a** and **b**



Draw a line with one endpoint coincident with the midpoint of **ca** and the other coincident with **bc**.

Label this line **ef** as shown.

Ensure that the point f does not pick up an automatic relation with the midpoint of cb

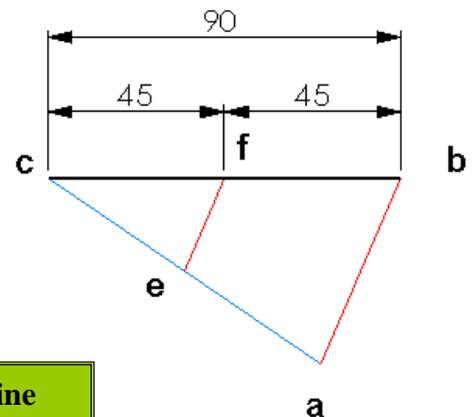


Add a parallel relation between **ef** and **ab**.

Smart dimension the distance **cf** and **fb**

|cf| = |fb| Why?

e is the midpoint of **ca**. Therefore **ca** is divided in the ratio of 1:1.



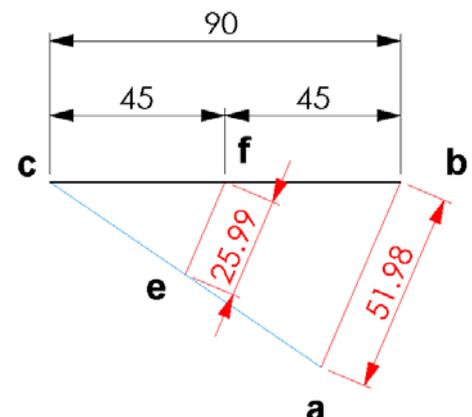
Two sides of a triangle are divided proportionally by a line drawn parallel to the third side.

ef is parallel to **ab**. Therefore **cb** will be divided in the same ratio, 1:1, two equal parts.

Exercise – Smart Dimension **ef** and **ab**.

Note: $ef = ab/2$

Therefore $\triangle cef$ is similar to $\triangle cab$

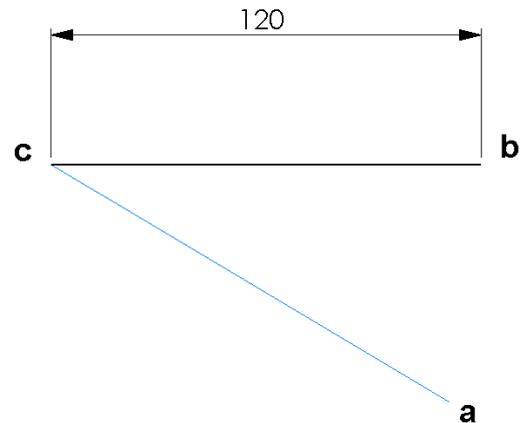


How can this principle be used to divide a line into 4 equal parts?

Previously we divided a line into two equal parts.

A line from the midpoint of one side of the triangle, dividing that side into two equal parts, was created parallel to the base, thereby dividing the other side in the same ratio.

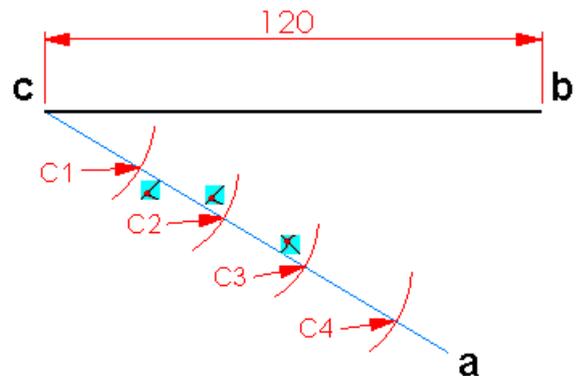
Create the sketch shown opposite.



To divide the line **bc** into 4 equal parts we must first create a triangle with one side divided in that ratio.

To achieve this we will sketch four Centrepoint Arcs on line **ca**. The centre of the first arc is coincident with **c**.

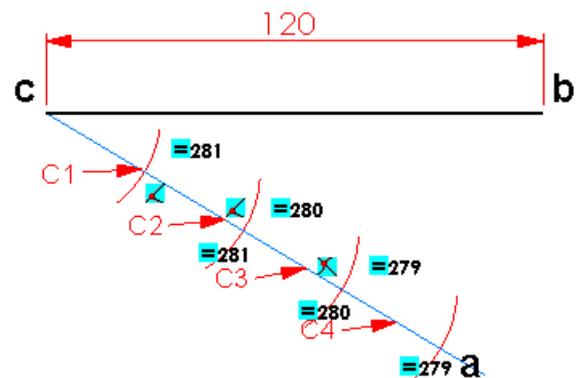
The centre of the next circle is coincident with the intersection of the previous arc and the line **ca**.



Select the 4 arcs and add an equal relation between them.

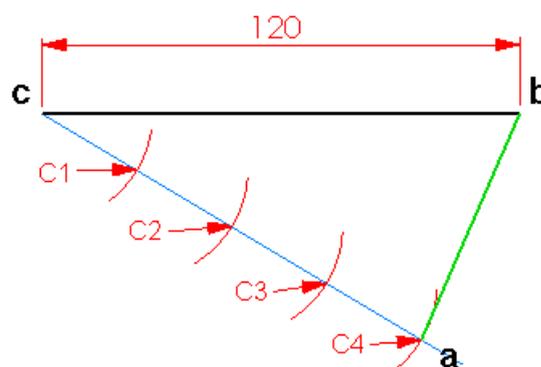
This ensures that $|cc_1| = |c_1c_2| = |c_2c_3| = |c_3c_4|$

The line **cc₄** is divided into 4 equal parts

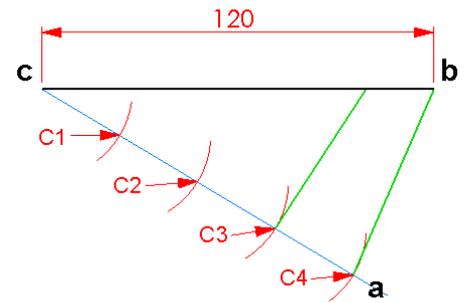


Join **c₄** to endpoint of **cb**

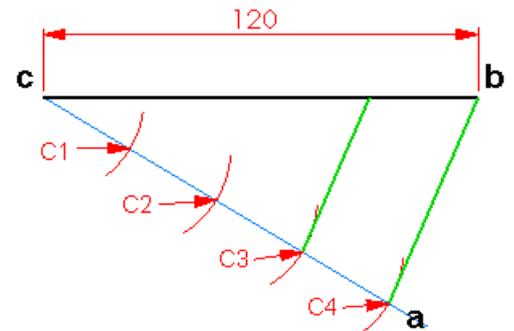
A triangle is created, **cc₄b**, with one side, **cc₄**, divided into 4 equal parts



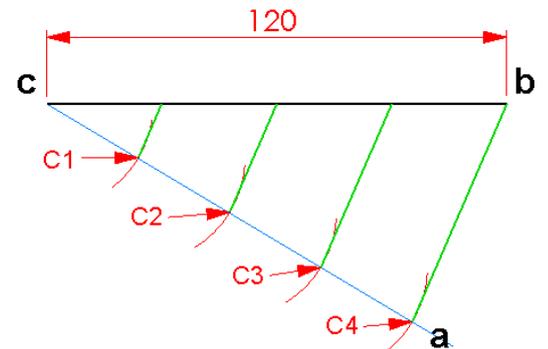
Sketch a line from c_3 , with the other endpoint **coincident** with ab .



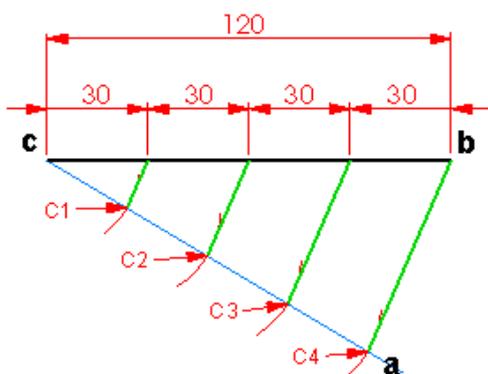
Add a parallel relation between the c_4b and this line.



Repeat the same procedure for c_1 and c_2



Smart dimension the points on the line cb , as shown.



The line cb is divided into the same ratio as cc_4 , 4 equal parts. Why?

Two sides of a triangle are divided proportionally by a line drawn parallel to the third side.

How can a line be divided into a particular ratio?

What is ratio?

Pictured are 5 cakes.



3 cakes are given to one individual and 2 to another



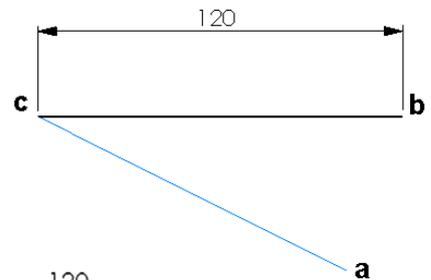
The 5 cakes are said to be divided in the ratio of 3:2 between the two individuals

Dividing a line in a ratio is approached in the same way as equal division of lines.

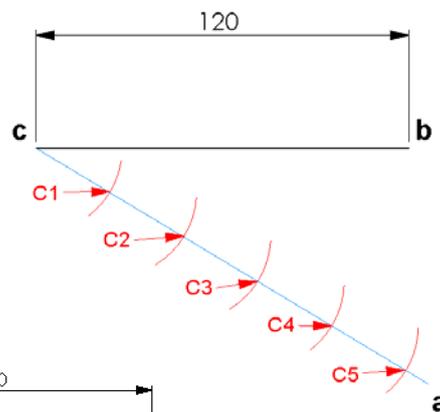
Count the number of divisions required by adding the ratio.

The number of divisions equals $3 + 2 = 5$

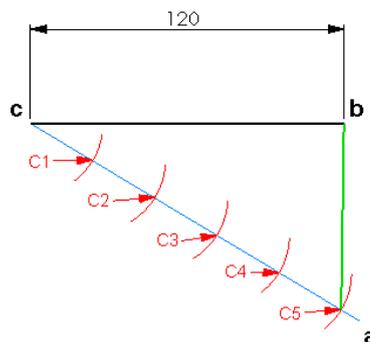
Create the sketch shown opposite.
Smart dimension the line $cb = 120\text{mm}$



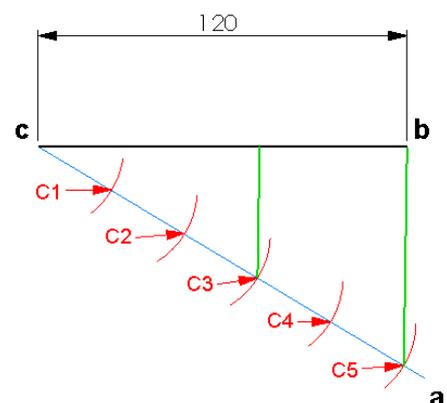
Mark 5 equal divisions along the line ca using the same method as before.



Sketch a line coincident with c_5 and b .



Add a line with endpoints coincident with c_3 and line cb



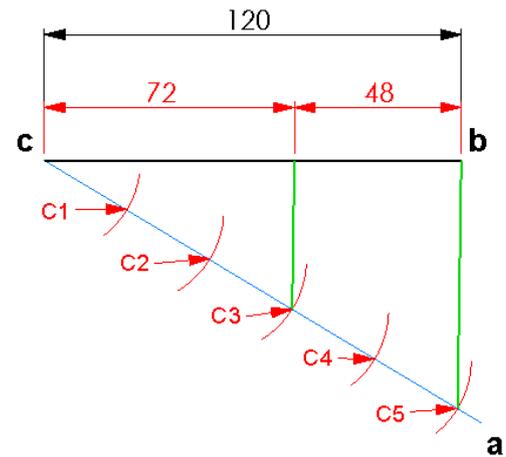
Add a parallel relation between c_5b and the line through c_3

Smart dimension $|cd|$ and $|db|$

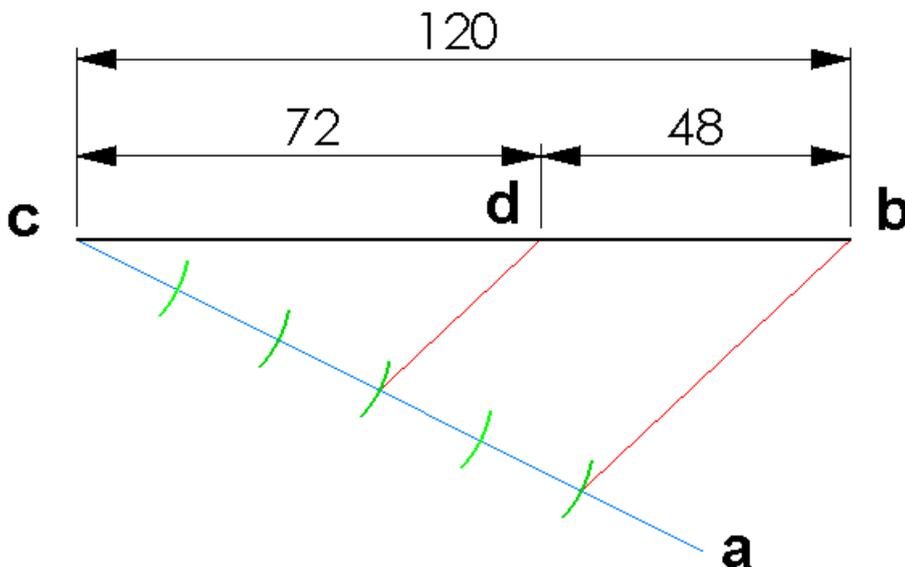
$$|cd| = 72\text{mm} \quad |db| = 48\text{mm}$$

$$cd/db = 72/48 = 3/2 \quad \text{or } 3:2$$

The line cb has been divided in the ratio of **3:2**



In practice the circles will not be drawn completely and the construction will appear as shown below.



Circles in Contact

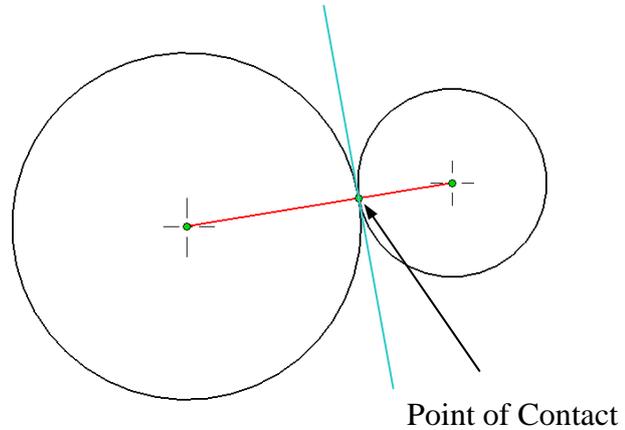
When two circles touch each other at only one point, they are in **tangential contact**. The point common to both circles is called the **Point of Contact (POC)**.

Circles in contact may be described as touching **internally** or **externally**.

Two circles touch **externally** when one is outside the other.

The point of contact is contained on a line joining the two centres

This point of contact is also contained on the tangent which is common to both circles.

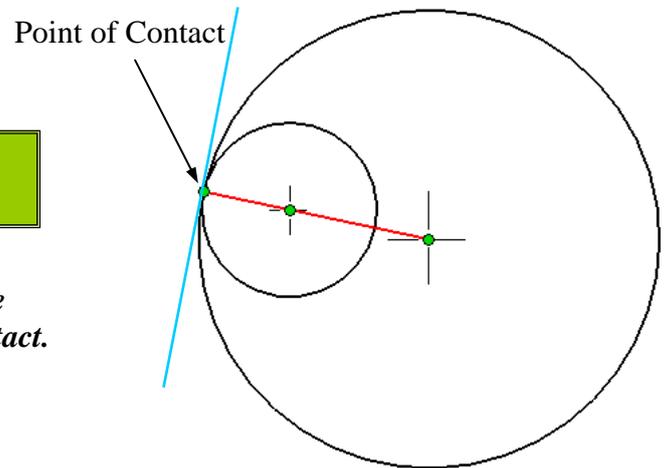


Two circles touch **internally** when one is inside the other.

The point of contact is contained on a line joining the two centres

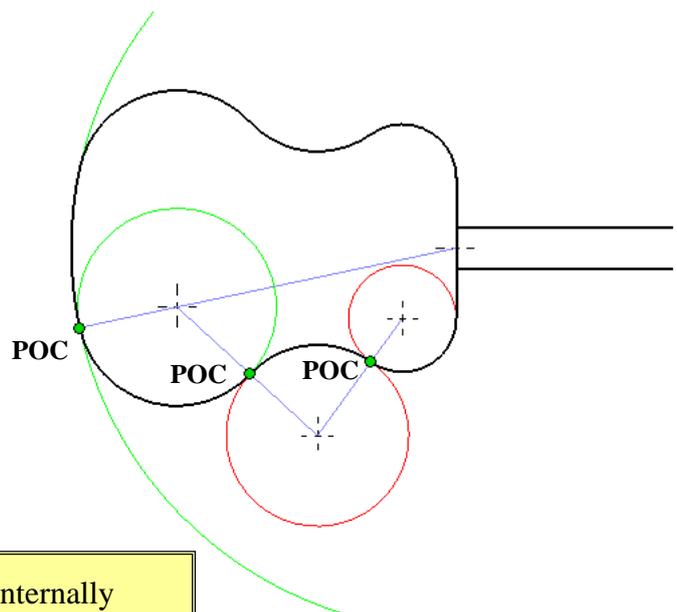
Note: When two circles touch internally the line joining the two centres must be extended to determine the point of contact.

This point of contact is also contained on the tangent which is common to both circles.



There are various examples of circles in contact in the world we live in.

The acoustic guitar is just one example of this.

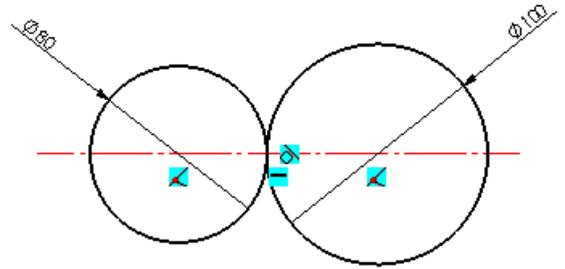


Can you identify the circles touching internally and the circles touching externally in this example?

Circles touching externally

Create the sketch shown opposite ensuring that the centres are coincident with the centre line and the circles are in tangential contact.

Smart dimension the two circles $\varnothing 80\text{mm}$ and $\varnothing 100\text{mm}$.



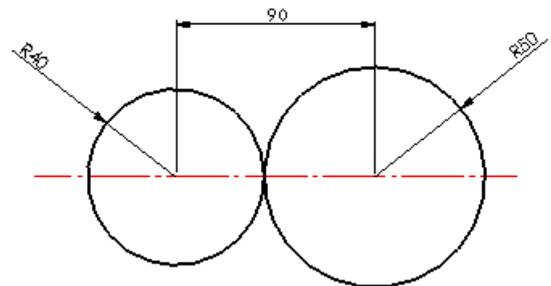
Right click on the dimension, select **Display Options, Display As Radius**
Repeat for the other circle.

The circles are now dimensioned **R40** and **R50**.
The radius is half the diameter.

Smart dimension the distance between centres.

The distance between centres is **90mm**.

$$40 + 50 = 90$$



When two circles touch externally, the distance between their centres is equal to the sum of their radii

Circles touching internally

Create the sketch shown opposite ensuring that the centres are coincident with the centre line and the circles are in tangential contact.

Smart dimension the two circles $\varnothing 180\text{mm}$ and $\varnothing 120\text{mm}$.

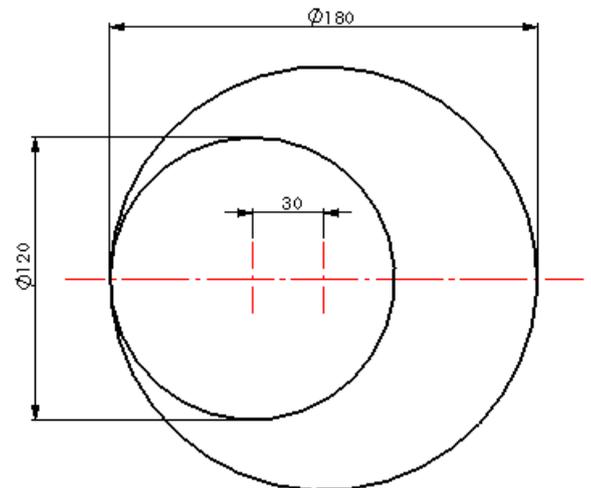
Display the dimensions as radii.

Smart dimension the distance between centres.

The larger circle is **R90mm** and the smaller is **R60mm**.

The distance between centres is **30mm**.

$$90 - 60 = 30$$

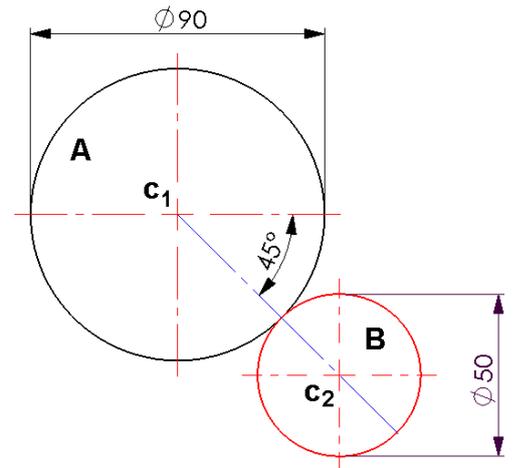


When two circles touch internally, the distance between their centres is equal to the difference of their radii

Circles touching externally

A problem is presented with two circles, $\varnothing 90\text{mm}$ and $\varnothing 50\text{mm}$, touching externally.

Using the principles established we will reproduce the figure shown.

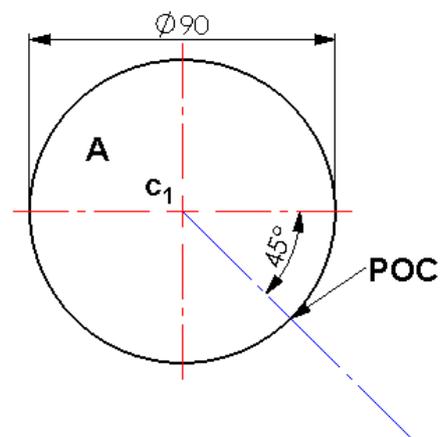


Where do we start?

Sketch circle **A**. Smart dimension the circle 90mm.

Add the blue line from c_1 and smart dimension the line at an angle of 45° to the horizontal. c_2 is contained on this line

Where this line cuts the circumference of the circle yields the **point of contact**.



The point of contact is contained on a line joining the two centres

How do we locate c_2 ?

When two circles touch externally, the distance between their centres is equal to the sum of their radii

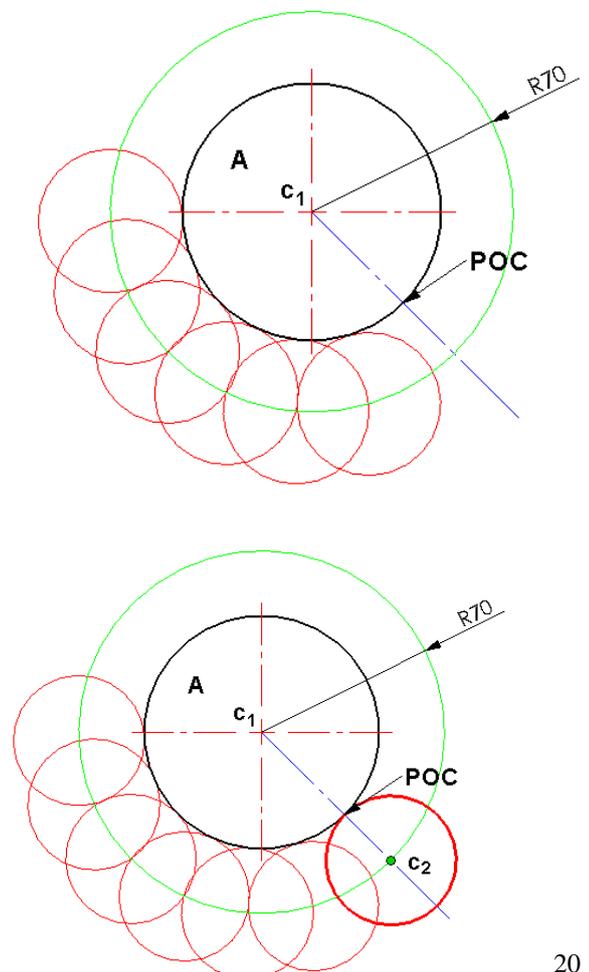
Add the two radii together – $45 + 25 = 70$

Sketch a circle of centre c_1 and radius **70mm**.

Any radius 25mm circle drawn with its centre positioned on the circumference of the green circle will be tangential to the black circle. Only one will be tangential at **POC**

The R70mm circle is the path of the centre of the circle B as it rolls around circle A. This path is referred to as a **locus**

The centre of B is located where the locus intersects the extension of the line joining c_1 and POC

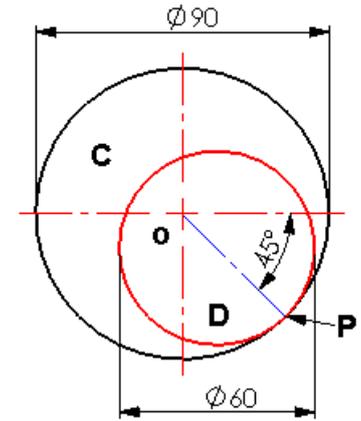


Circles touching internally

In this case two circles are presented, $\varnothing 90$ and $\varnothing 60$ in tangential contact.

However, in this case one circle is contained within the other. They are touching internally.

Using the principles established we will reproduce the figure shown.

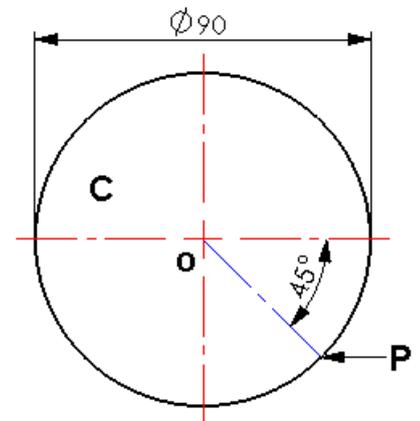


Sketch circle **C**. Smart dimension the circle 90mm.

Create a line, one end coincident with the centre of circle **C** and the other coincident with the circle circumference.

Locate the exact location for point **P** by smart dimensioning the angle 45° .

P is the point of contact between the two circles. Therefore, a line joining **P** to **o** will contain the centre of circle **D**



The point of contact is contained on a line joining the two centres

Secondly;

When two circles touch internally, the distance between their centres is equal to the difference of their radii

Find the difference between the two **radii** – $45 - 30 = 15$

The two circle centres will be 15mm apart.

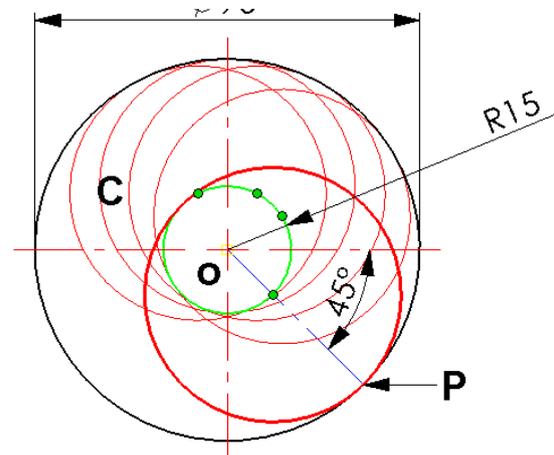
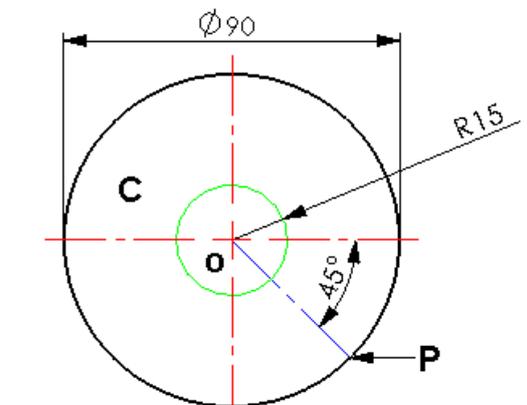
Sketch a circle with centre **O** and radius **15mm**

This circle represents the locus of the centre of circle **D** as it rolls around circle **C**.

Any radius 25mm circle drawn with its centre positioned on the locus will touch circle **C** internally.

Only one will be tangential at **POC**

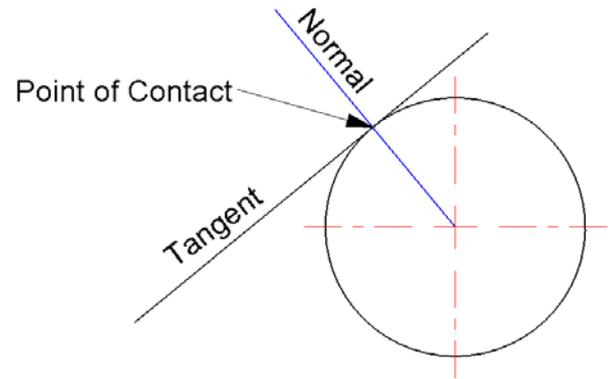
The centre of D is located where the locus intersects the line joining o to the point of contact.



Tangents to a Circle

A **tangent** to a circle is a line that touches the circumference at one point only, called the **Point of Contact**

A line drawn from the centre of the circle to the point of contact is called the **normal**.



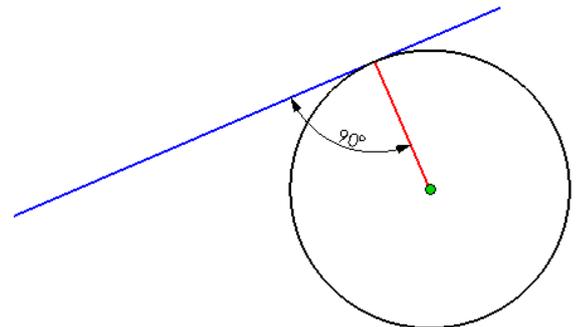
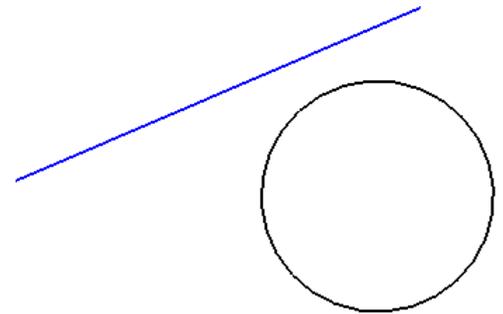
Lets investigate the relationship between the tangent and the normal.

Create the sketch shown opposite

Apply a **tangential relation** between the line and the circle.

Sketch the **normal**, a line joining the centre of the circle to the point of contact.

Smart dimension the angle between the normal and the tangent

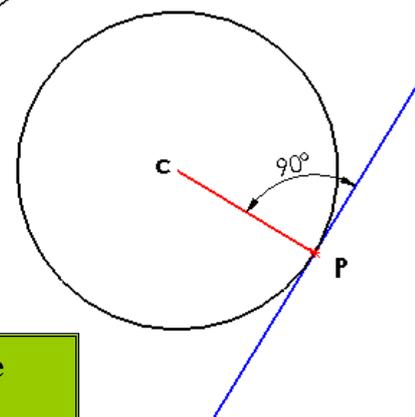
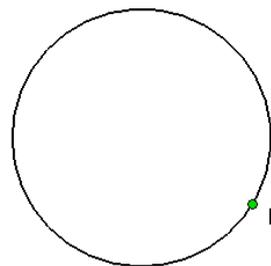


The angle between the tangent and the normal is always 90°

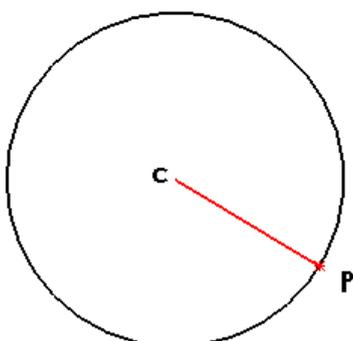
Generating a tangent to a circle given the point of contact

Given a circle A and a point of contact P on the circumference to generate a tangent to the circle

Sketch the normal by joining **p** to the centre of the circle, **c**



Generate the tangent by drawing a line perpendicular to **cp** through **p**.

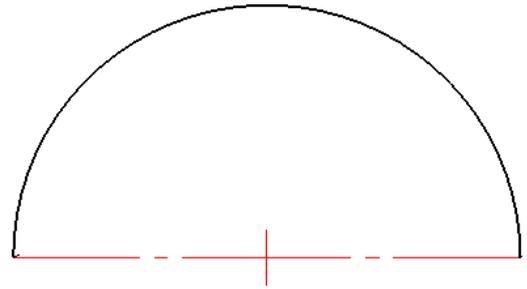


The angle between the tangent and the normal is always 90°

The construction for creating a tangent to a circle from a point outside the circle is based on one of the geometric principles of a circle.

Using SolidWorks we will explore that principle.

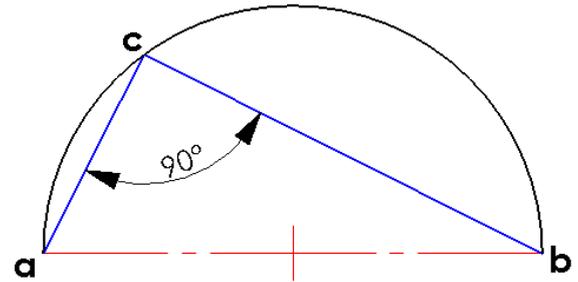
Create the semicircular sketch shown opposite.



Add the lines shown coincident with **a** and **b** and the circumference of the semicircle, **c**.

Smart dimension the $\angle acb$

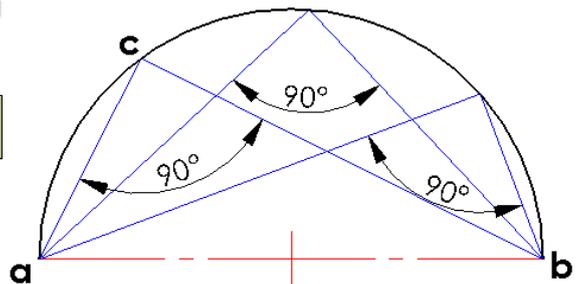
The angle generated is 90° .



Generate further angles on the diameter with **c** coincident with the circumference.

Smart dimension these angles also.

The angle standing in a semicircle is always 90°



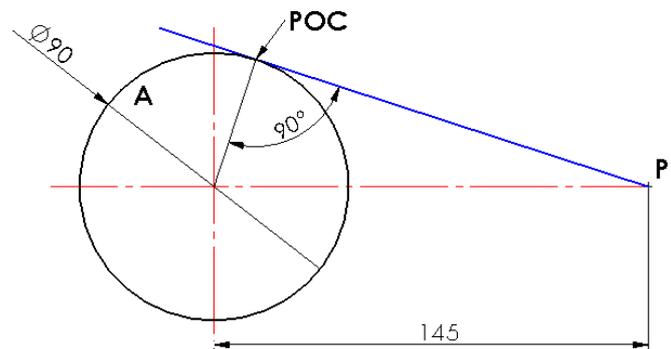
How can this be useful in constructing a tangent to a circle?

 What angle does a tangent make with a normal?
 90°

Tangent to a circle from a point outside the curve

A problem is presented with one circle, $\varnothing 90\text{mm}$ and a tangent, touching the circle from a point **P** outside. The point of contact is not known

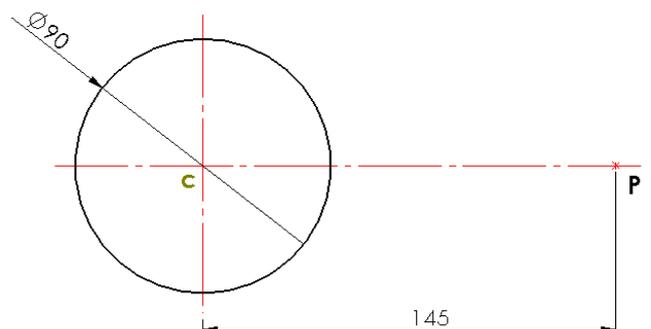
Using the principles established we will reproduce the figure shown.



Where do we start?

Create the sketch shown opposite.

Ensure that the centre lines intersect at the centre of the circle and point **P** is coincident with the horizontal centreline.

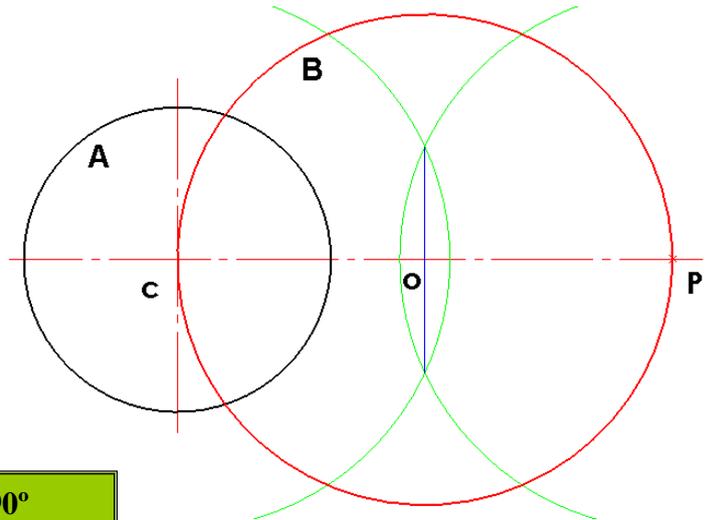


We will now generate a circle with $|cp|$ as diameter

Bisect the line cp to find the midpoint o

Sketch a circle **B** with centre o and radius $|oc|$

Any point, chosen on the circumference of the circle, joined to points c and p will generate a 90° angle.



The angle standing in a semicircle is always 90°

Join points c and p to point x , the intersection of circles **A** and **B**.

Smart dimension $\angle cxb$.

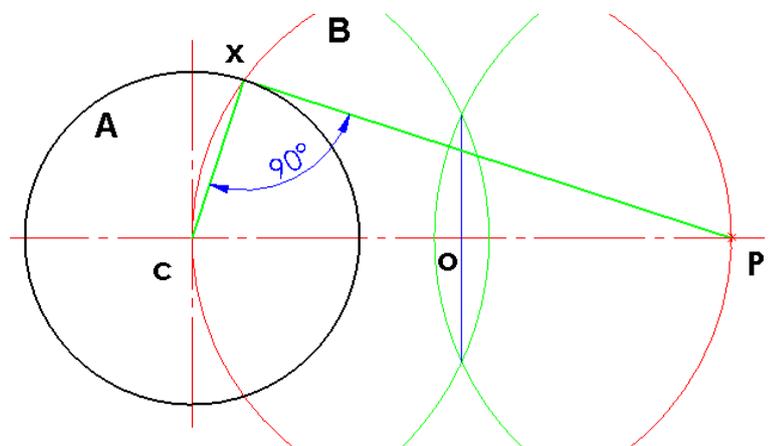
$\angle cxb = 90^\circ$

x is the point of contact

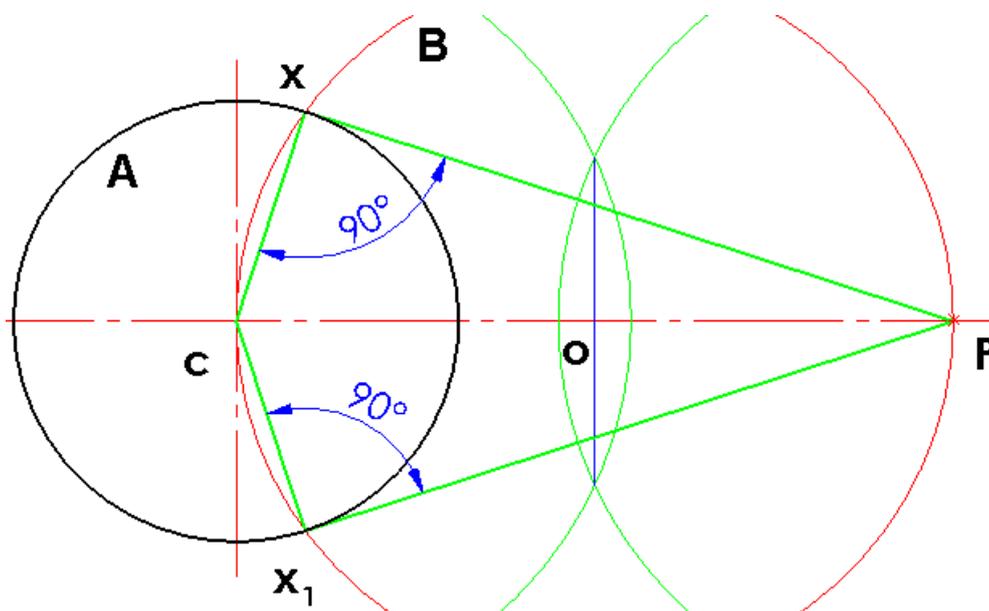
cx is a normal from the centre of the circle

xp is perpendicular to cx

therefore, xp is tangential to the circle **A**.



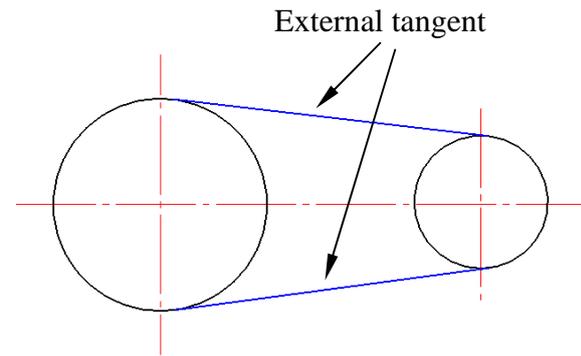
Note: If a line is drawn from P to x_1 a second tangent will be formed.



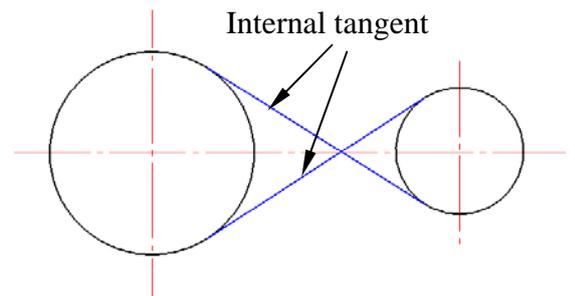
Common tangents between circles

There are 4 tangents which may be drawn between the given circles.

When the tangent created does not intersect the line segment joining the two centres it is referred to as an **External Tangent**

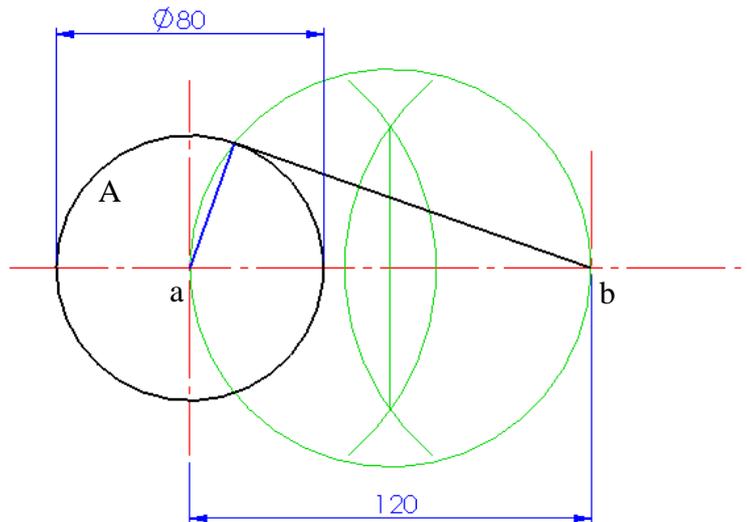


When the tangent created intersects the line segment joining the two centres it is referred to as an **Internal Tangent**



The construction to determine the points of contact and normal in this case uses the principle used for creating a tangent a circle and a point outside the circle.

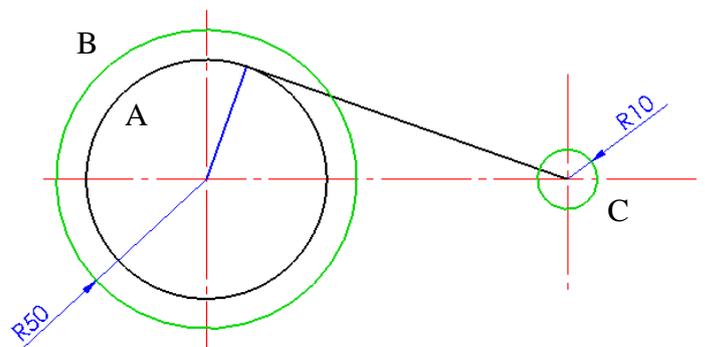
Create the sketch shown opposite based on the previous construction.



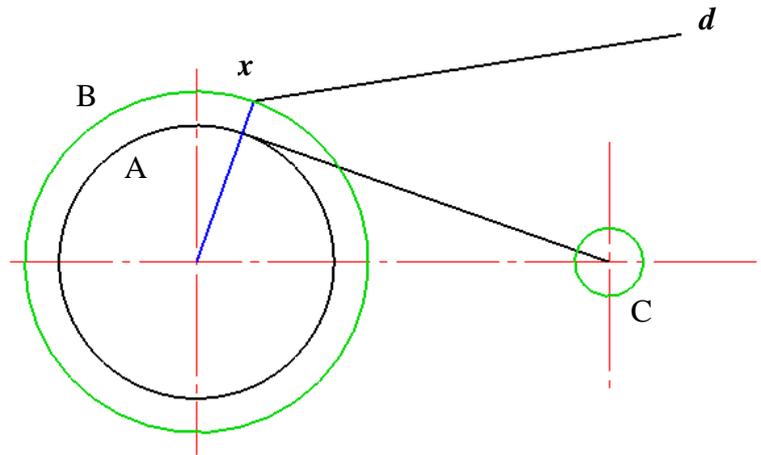
Add a circle **B** of $\text{Ø}100$ concentric with circle A.
Create another circle **C** with centre b and radius 10mm.

The radial difference between A and B is equal to the radius of C; $50 - 40 = 10$ (Radius of C)

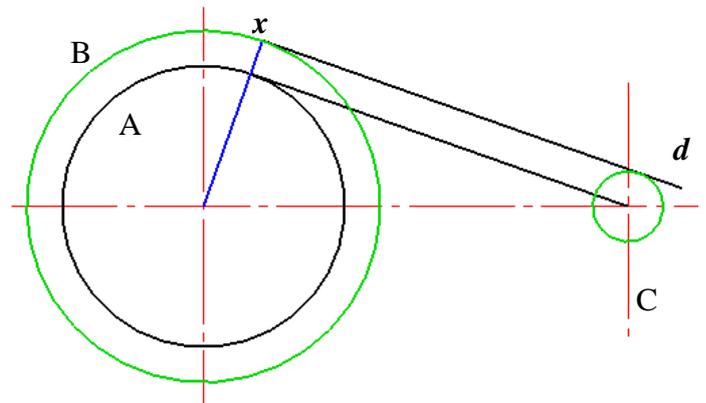
(Construction has been omitted for clarity)



Extend the normal to intersect circle **B** at **x**.
Draw a line from **x** as shown.

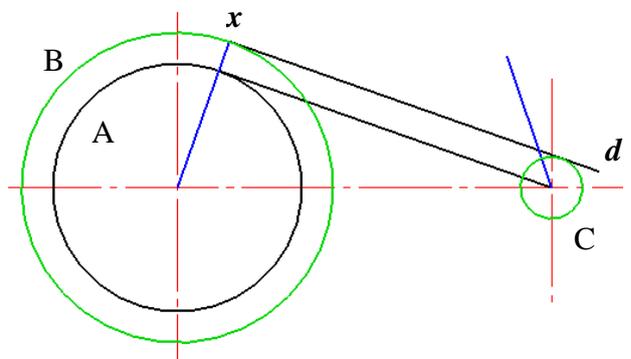


Add a parallel relation between the line **xd** and the tangent.



The new location of the line **xd** will be a common tangent to circles **B** and **C**.

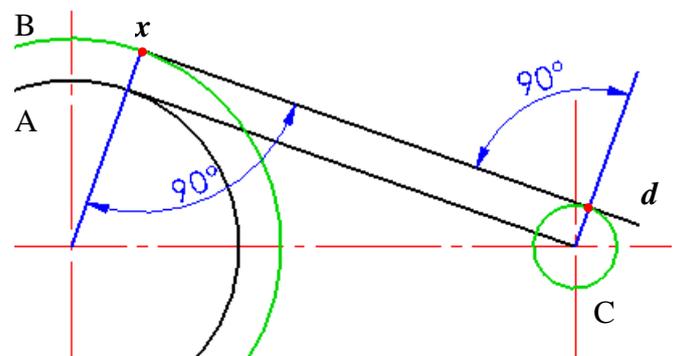
Add the line shown passing through the centre of **C**.



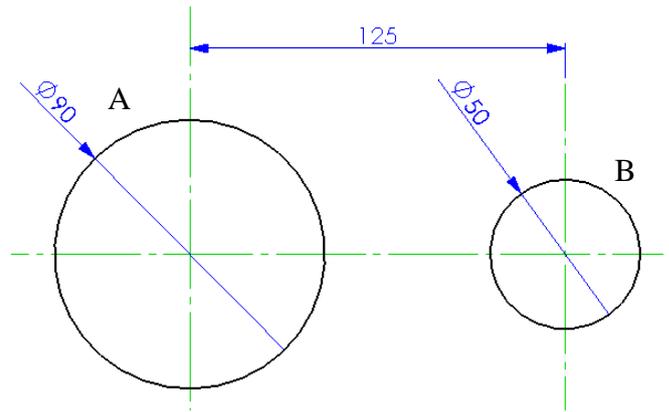
Add a **parallel relation** between the two blue lines and Smart dimension the angle they create with the line **xd**.

The two lines form a right angle with the tangent and their intersection with the tangent indicates the position of the **points of contact**.

The two blue lines are **normals** to the tangent.

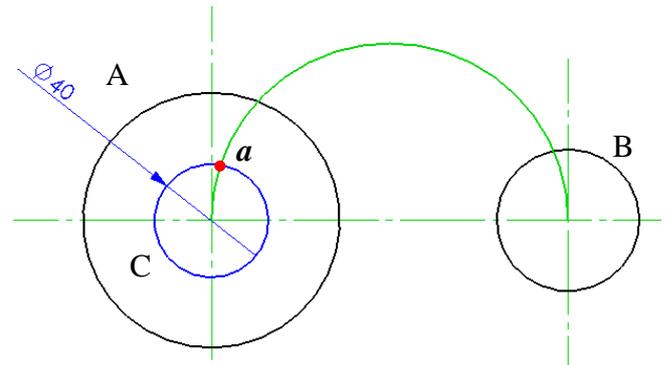


Activity – Given the two circles shown, to construct a common external tangent between the two circles.

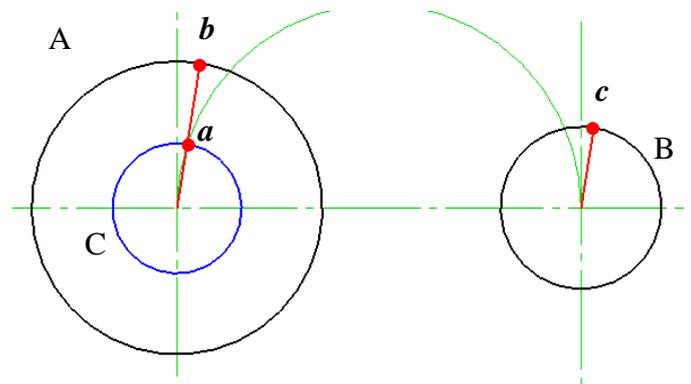


Create circle **C** with radius equal to the difference between the radii of A and B.

Radius of C = Radius of A - Radius of B, $45 - 25 = 20$



Create a line joining the centre of **A** and **a**.
Extend this line to intersect circle A at **b**. This is a normal to the tangent and yields the point of contact of the tangent with A.



Draw another line through the centre of B parallel to this normal. This yields the point of contact **c** with B.

Join points **b** and **c** – the common external tangent between A and B

Create a mirror image of **b** and **c**, using the line segment joining their centres as the axis, to yield points **d** and **e**.

Points **d** and **e** are the points of contact of the other common external tangent.

